The Basics of Filament Stretching Rheometry

Introduction

Measurement of the elongational behavior of fluids is important both for basic research purposes and in industrial applications, since many complex flows contain strong extensional components, including extrusion flows, coating flows, contraction flows, and fiber spinning flows.

Several different experimental techniques have been developed over the years to measure elongational viscosities of non-Newtonian fluids. Due to non-ideal flows and unknown preshear histories in such devices as opposed jet rheometers, spin-line rheometers, and contraction flows, interpretation of results from these devices is quite difficult. Each of these devices yields at best an “apparent” elongational viscosity. Other devices, primarily filament stretching devices and two-and four-roll mills (Meissner Apparatus), are able to stretch polymer melts homogeneously and thus can provide more detailed information about extensional viscosities via measurements of force and cross-sectional area of the sample [1-3].

Measurement of elongational properties of ‘*mobile*’ fluids such as dilute polymer solutions has been notoriously difficult, and the filament stretching extensional rheometer has emerged as one of the most controllable and detailed methods of measuring transient extensional flow behavior of these fluids. An early filament stretching device was introduced in 1990 by Matta and Tytus [4], and the modern filament stretching rheometer was then developed by Sridhar and coworkers based on this early device [5]. This application note will demonstrate the use of the filament stretching rheometer and the typical kinematics and apparati used to make these measurements [6-8].

Theory

In order to probe the extensional behavior of a fluid, one desires to impose a flow that isolates extensional components from shearing components. One simple ‘shear-free’ flow, *homogeneous pure uniaxial elongation*, is described by the following velocity field,

$$\begin{aligned} v_r &= -\frac{1}{2}\dot{\varepsilon}_0 r \\ v_\theta &= 0 \\ v_z &= +\dot{\varepsilon}_0 z \end{aligned} \tag{1}$$

The strain rate tensor for this flow contains only diagonal components, and thus the *extension rate* $\dot{\varepsilon}_0$ is a constant. In this flow field, local fluid elements move apart exponentially in time, such that the distance separating two material elements is described by

$$\Delta\ell = \ell_0 e^{+\dot{\varepsilon}_0 t} \tag{2}$$

In uniaxial elongational flow, the desired material function is the *transient extensional viscosity* as a function of time and commanded elongation rate, $\dot{\varepsilon}_0$. This transient extensional viscosity is defined in terms of the tensile stress growth in the fluid as a function of the imposed deformation rate in the fluid filament, according to the expression

56 Roland Street, Suite 310 · Boston, MA 02129 · Office (617) 629-4400 · Fax (617) 629-9100 · info@campoly.com ·

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$$\overline{\eta}^{+}(t,\dot{\varepsilon}_0) = \frac{[\tau_{zz}(t)-\tau_{rr}(t)]}{\dot{\varepsilon}_0} \tag{3}$$

The filament stretching device attempts to create the purely uniaxial elongational flow described above by placing a small sample of fluid between two circular endplates, as shown schematically in Figure 1. The endplates are then moved apart so that the gap between the plates increases exponentially in time, satisfying

$$L_p(t) = L_0 exp\left(\dot{E}t\right) \tag{4}$$

In this experiment, the *axial stretch rate* is defined by the velocity of the endplates,

$$\dot{E} = \dot{L}_p \big/ L_p \tag{5}$$

and is constant, where the ($^{\cdot}$) indicates a derivative with respect to time. The *Hencky strain*, typically used as a dimensionless time in comparing transient extensional rheology profiles, is defined in terms of the separation of the endplates by

$$\varepsilon_p = \dot{E}t = ln\left[L_p(t)/L_0\right] \tag{6}$$

However, the actual flow created in the filament stretching device is not quite ideal, since the no-slip boundary condition causes an additional shearing component of flow to be induced near the endplates. For small strains, the actual flow closely approximates a 'reverse' squeeze flow that can be accurately represented by a lubrication analysis.

The quantities measured in a filament stretching device include the total force exerted on the endplate, F_p(t), and the mid-filament radius, R_{mid}(t), as shown in Figure 1. In experiments, the mid-filament radius profile is observed to decrease in a complicated way due to the no-slip boundary condition. As a result, fluid elements near the midplane of the filament experience a different deformation rate than those near the endplates. Since fluid elements near the midplane experience a more nearly shear-free flow, the transient extensional viscosity is often computed based on the *effective deformation rate* $\dot{\varepsilon}_{eff}$, given by

$$\dot{\varepsilon}_{eff}(t) = -\frac{2}{R_{mid}}\frac{dR_{mid}}{dt} \tag{7}$$

In addition, the *effective Hencky strain* in the fluid filament is now computed via

$$\varepsilon_{eff} = -\int \frac{2}{R_{mid}}\frac{dR_{mid}}{dt}dt = 2ln(R_0/R_{mid}) \tag{8}$$

By performing a force balance on the filament, the transient extensional viscosity can be related to the measured and computed quantities discussed above. The transient extensional viscosity is frequently expressed in non-dimensional form as the *Trouton ratio* (*Tr*), or the ratio of the transient extensional viscosity to the zero-shear viscosity in the fluid, η_0. Thus, the transient Trouton ratio is related to measured quantities by

$$Tr = \frac{\overline{\eta}^{+}(t,\dot{\varepsilon}_0)}{\eta_0} = \frac{[\tau_{zz}(t)-\tau_{rr}(t)]}{\dot{\varepsilon}_0} = \frac{F_{p(t)}}{\pi R_{mid}^2(t)\boldsymbol{\eta_0}\dot{\varepsilon}_{eff}(t)} \tag{9}$$

Apparatus

Figure 2 shows a schematic diagram of the custom build filament stretching device used in our laboratories. The apparatus is based around a motion control system consisting of two linear DC brushless motors moving along the same axis. The upper and lower endplate assemblies are mounted to the upper and lower motors, respectively. The bottom endplate is then mounted to a sensitive force transducer, and a CCD laser micrometer, used to measure the mid-filament radius, is mounted at a fixed point along the axis of the motion.

The operating space of a filament stretching rheometer can be conveniently represented by plotting the velocity of the plates against the plate separation. Since the plate separation in a filament stretching experiment grows exponentially in time, a single experiment can be represented as a straight line on this operating diagram. The filament stretching rheometer used in this laboratory has an operating space, shown in Figure 3, capable of achieving Hencky strains up to $\varepsilon_f \approx 6.5$ and elongational stretch rates of $0.1\ s^{-1} \leq \dot{\varepsilon}_0 \leq 20\ s^{-1}$. These limits were estimated based on an initial plate separation of L_0 = 0.25 cm, and an endplate radius of R_0 = 0.35 cm.

All data relevant to the transient extensional rheology of the stretching filament, including total force on the endplate, mid-filament radius, plate positions, and position errors, are gathered synchronously and in real-time by the motion control system. Video images of the deforming filament are simultaneously recorded in the translating frame of the bottom endplate. During post-processing, raw force data is deconvolved from the response function of the load cell and the processed data is then filtered. The *logarithm* of the raw mid-filament radius data is fit to a low-order polynomial function, and the effective deformation rate, $\dot{\varepsilon}_{eff}(t)$, is computed via the fitted polynomial function and equation (7). Finally, the transient Trouton ratio is computed as a function of Hencky strain via equations (8) and (9).

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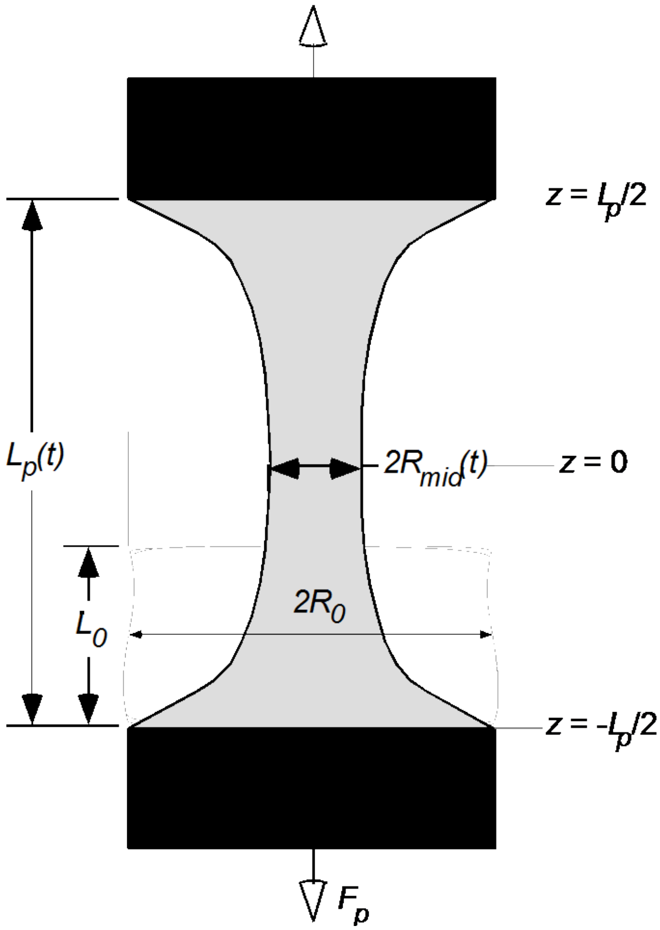

Figure 1. Sketch of a filament stretching apparatus. The origin is taken to be at the midplane of the filament.

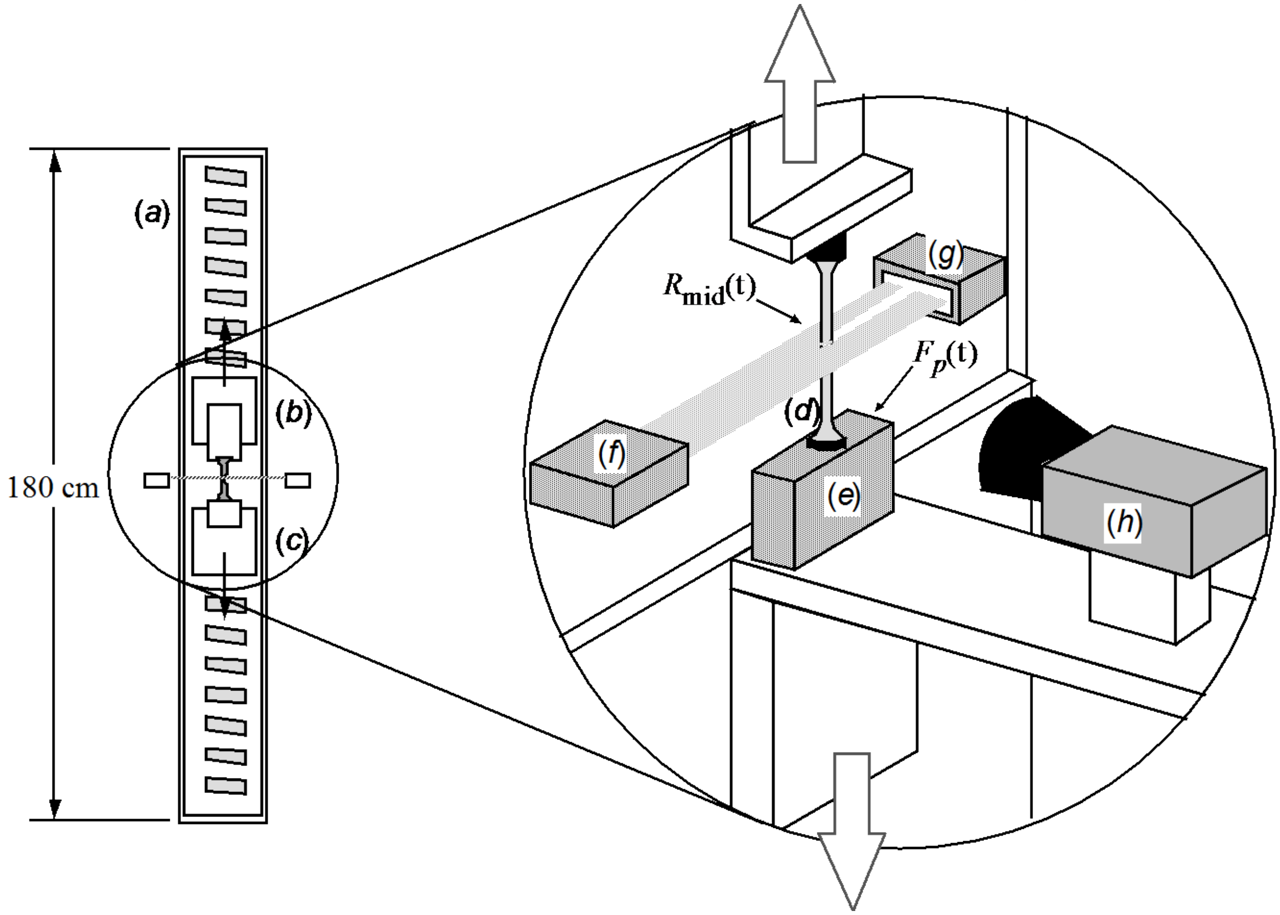

Figure 2. Schematic diagram of filament stretching rheometer: (*a*) linear DC brushless motor; (*b*) upper motor, with top endplate assembly; (*c*) lower motor, with bottom endplate assembly; (*d*) fluid sample; (*e*) force transducer, (*f*) CCD laser micrometer, transmitter, (*g*) receiver; (*h*) CCD camera.

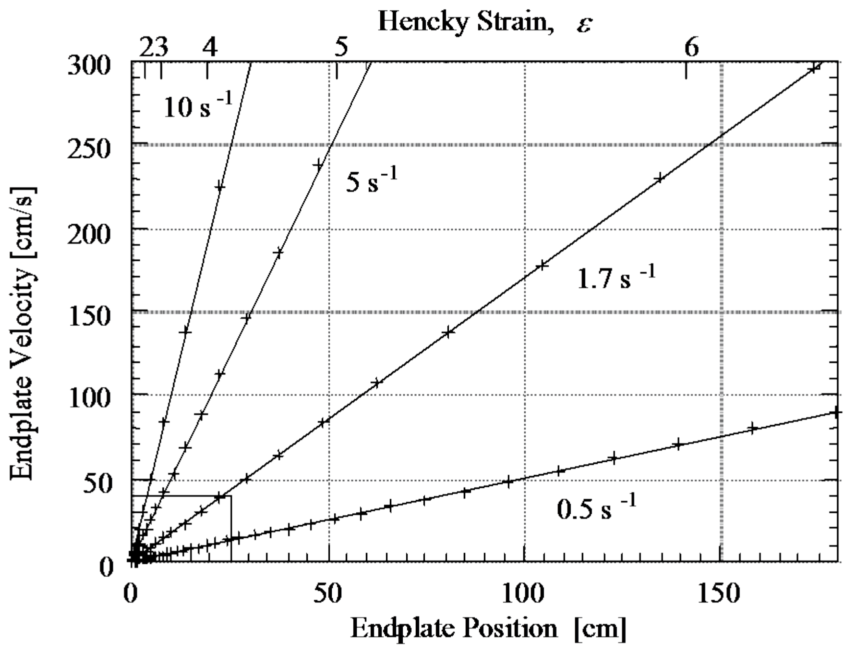

Figure 3. Operating diagram for filament stretching rheometer. This figure shows the range of velocities and endplate separations achievable, with corresponding Hencky strains shown along the top axis. The small box at lower left shows a similar operating space for a previous device.

For more information, contact:

Cambridge Polymer Group, Inc.
56 Roland St.
Boston, MA 02129
ph: (617)629-4400
http://www.campoly.com
info@campoly.com